Statistical methods for managing uncertainty in complex models and their application to global change science

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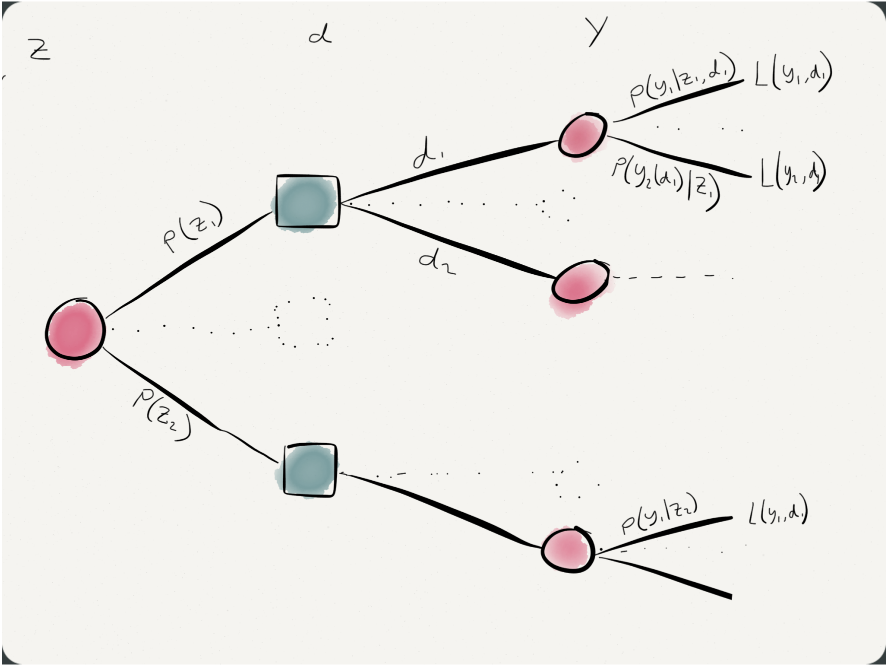

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p(z₂)
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p(y₂(d₁)|z₁)
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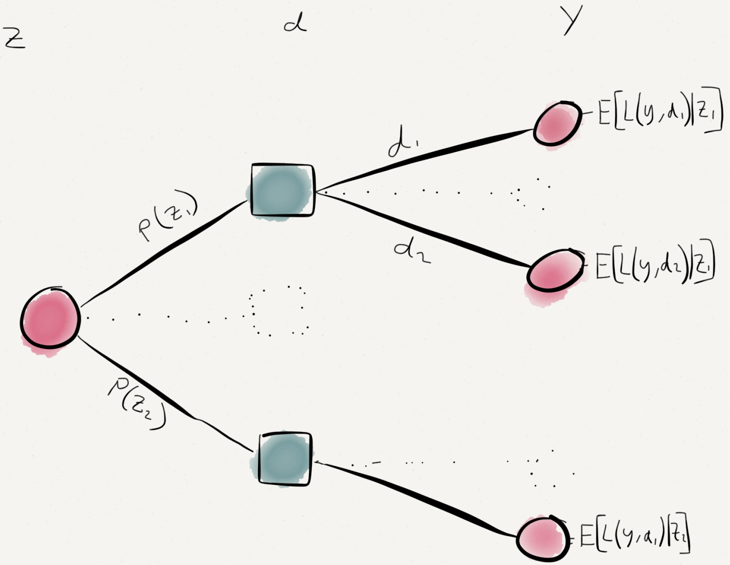
L(y₂,d₁)
p(y₁|z₂)
L(y₁,d₁)



z
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E[L(y,d1)|z1]
E[L(y,d2)|z1]
E[L(y,d1)|z2]

Reality, Data, Models and Loss: A framework

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We use models to assist with both.

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- If we can find $\mathrm{P}(Y)$ and we know the distribution of e_H, we can easily derive $\mathrm{P}(Y_F|Z_H)$.

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- If we can find $\mathrm{P}(Y)$ and we know the distribution of e_H, we can easily derive $\mathrm{P}(Y_F|Z_H)$.
- Enter climate models.
- We have a selection of climate models $f_i(x_{[i]}, \theta)$ used to try to predict Y under forcing θ.
- How can information from the f_i's get us to $\mathrm{P}(Y)$?

Statistical modelling

One model approach:

- Each model is informative for $Y(\theta)$, but there is structural discrepancy left over:

$$Y(\theta) = f_i(x^*_{[i]}, \theta) + \eta_i(\theta)$$

- We can get Monte Carlo samples from $\mathrm{P}(Y(\theta))$ if we can sample from

$$\mathrm{P}(f_i(x^*_{[i]}, \theta)|x^*_{[i]})\mathrm{P}(x^*_{[i]})\mathrm{P}(\eta_i(\theta))$$

- E.g. *Kennedy and O'Hagan (2001), Goldstein and Rougier (2004), Rougier (2007), Sexton et al (2011), Bhat et al. (2012), Chang et al. (2014)*

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Statistical modelling

Multi-model approach:

- The models are exchangeable and $Y(\theta)$ relates to the collection: E.g.

$$f_i(x^*_{[i]}, \theta) = \mathcal{M}(\theta) + R_i(\theta); \qquad Y(\theta) = \alpha\mathcal{M}(\theta) + U(\theta)$$

- We observe $f_1(x^t_{[1]}), \ldots, f_n(x^t_{[n]})$ and we can get Monte Carlo samples from $\mathrm{P}(Y(\theta))$ if we can sample from

$$\mathrm{P}(U(\theta))\mathrm{P}(\alpha, \mathcal{M}(\theta)) \prod_{i=1}^{k} \mathrm{P}(f_i(x^*_{[i]}) | f_i(x^t_{[i]}), x^*_{[i]}, \mathcal{M}(\theta))\mathrm{P}(x^*_{[i]})$$

- E.g. *Tebaldi et al. (2005), Tebaldi and Sanso, (2009), Chandler (2013), Rougier et al. (2013), Williamson et al. (2013)*

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Current practice: What lurks in the conditioning?

Often, uncertainties are ignored instead of quantified. This does not reduce uncertainty, it removes problems to the conditioning...

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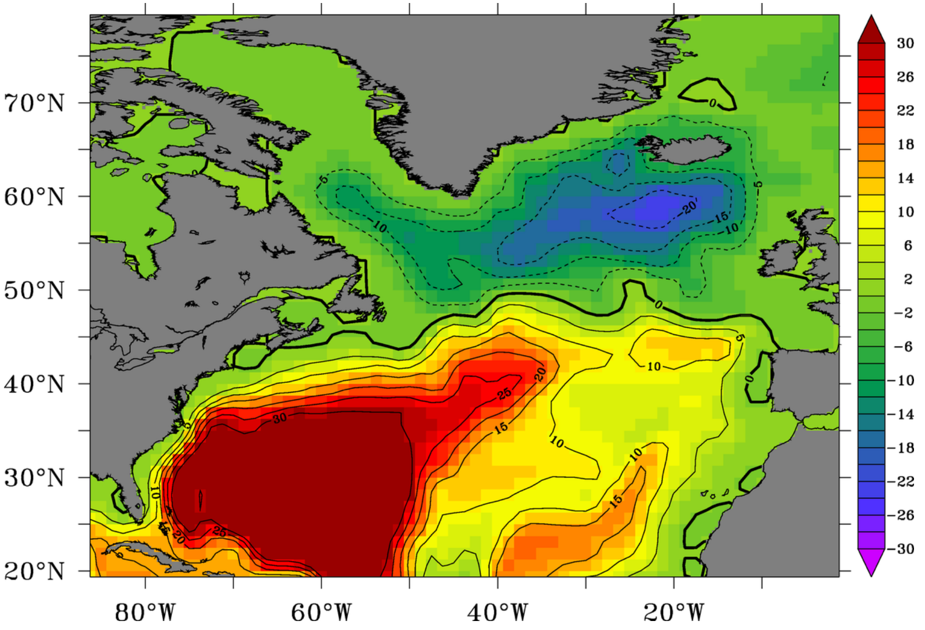
Example

- The CMIP GCMs are run at $x^t_{[i]} \neq x^*_{[i]}$. I.e. they are not optimally tuned.
- But this is rarely not addressed. In fact, we act as if $x^t_{[i]} = x^*_{[i]}$.
- Now $\mathrm{P}(x^*_{[i]})$ is gone and $\mathrm{P}(f_i(x^t_{[i]}, \theta))$, has no code uncertainty!
- Hence we obtain samples from internal variability only and can get to $\mathrm{P}(Y(\theta)|x^*_{[i]} = x^t_{[i]})$.
- Is this a called-off bet that we know is already called off?



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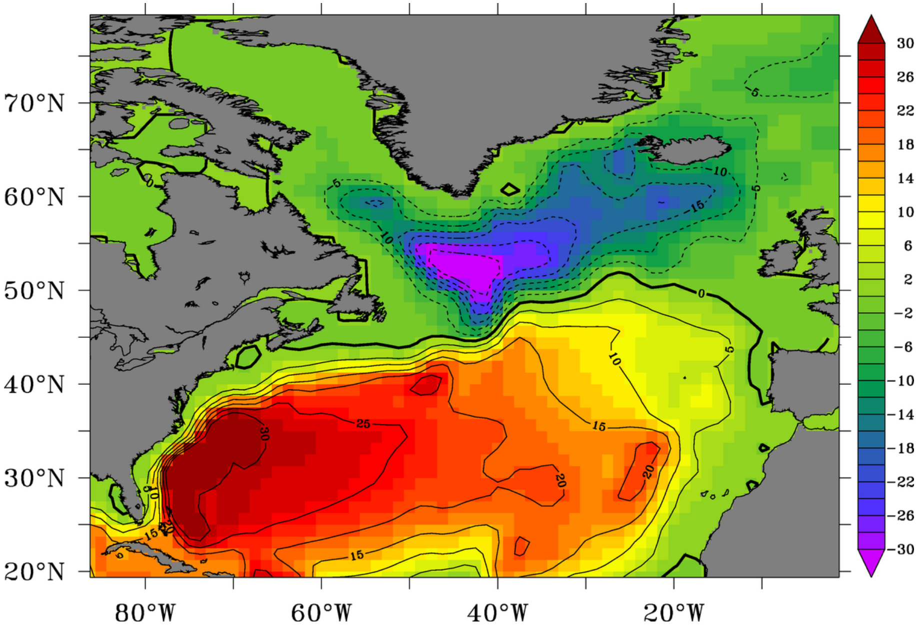

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25 years of statistical methods for complex models: an introduction

- Most scientific disciplines have computer models with parameters and quantifying parameter/code uncertainty is now well studied in statistics.
- We build a statistical model for the simulator that gives, for any x:
  1. A prediction at x
  2. Uncertainty on the prediction at x.

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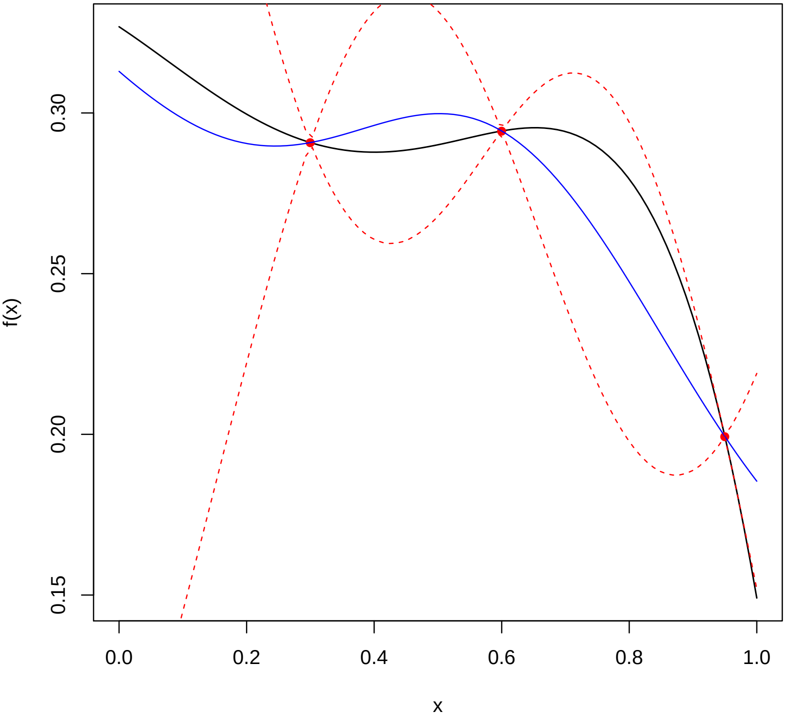
- If you've heard of Pattern Scaling, it is essentially a special case of an emulator (without 2 and with restrictions on the types of predictions allowed)
- *Haylock and O'Hagan (1996), Santner et al (2003), Rougier et al. (2009), Challenor et al (2009), Lee et al. (2011), Williamson et al. (2012), Williamson and Blaker (2014).*



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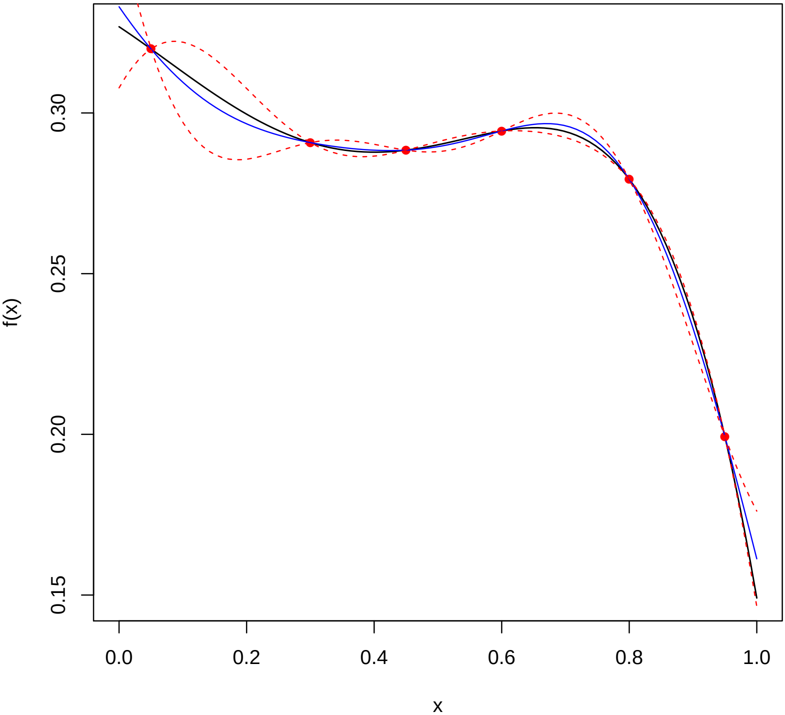
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Improving tuning: History matching and Calibration

- Emulators let us cheaply sample from $\mathrm{P}(f_i(x^*_{[i]}, \theta)|x^*_{[i]})\mathrm{P}(x^*_{[i]})$.
- They also allow us to improve tuning (reducing uncertainty in $\mathrm{P}(x^*_{[i]})$) to make models more informative.
- This happens by history matching:

$$\mathcal{I}(x)^2 = \frac{(Z - \mathrm{E}\left[f(x)\right])^2}{\mathrm{Var}\left[Z - \mathrm{E}\left[f(x)\right]\right]}.$$

- A point x_0 is ruled out of parameter space if $|\mathcal{I}(x_0)| > a$ for some threshold a
- *Vernon et al. (2010), Williamson et al. (2013), Edwards et al. (2011), Tokmakien and Challenor (2012), McNeall et al. (2013), Williamson et al. (2014).*

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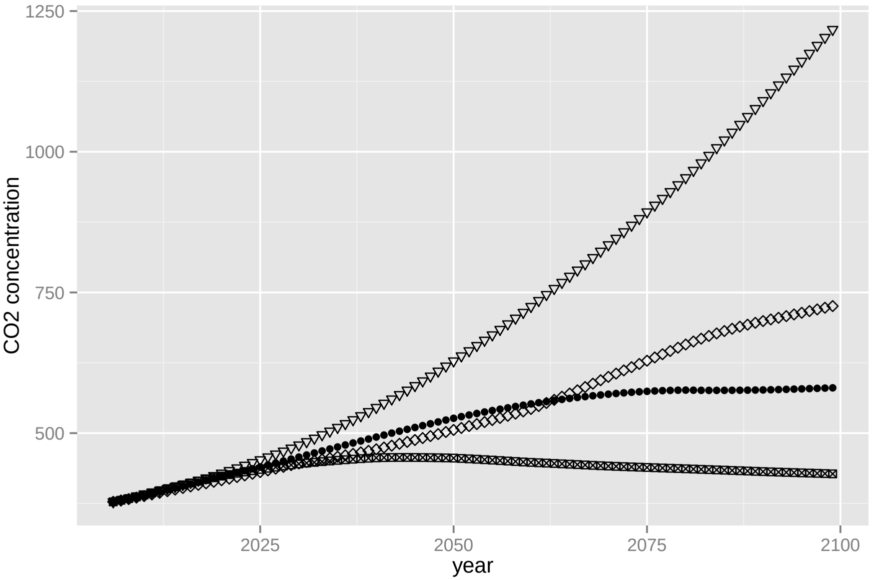
Policy support: Beyond Pattern Scaling

- $\mathrm{P}(Y(\theta)) = \mathrm{P}(Y|\theta)$.
- Can we get to $\mathrm{P}(Y)$ or $\mathrm{P}(Y|\theta^*)$?
- How do we make inference and provide decision support beyond the RCPs/SSPs?
- Emulation can help AND carry through the uncertainty!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CO2 concentration
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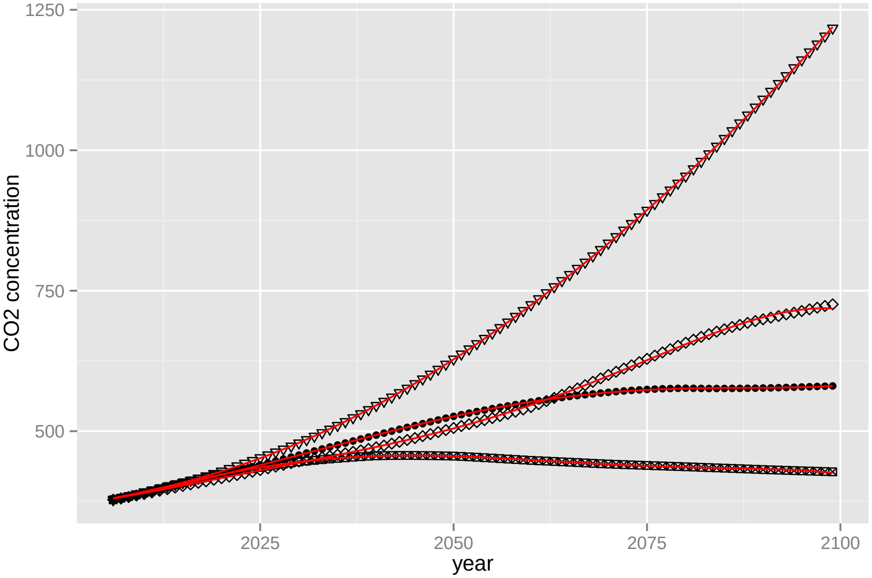

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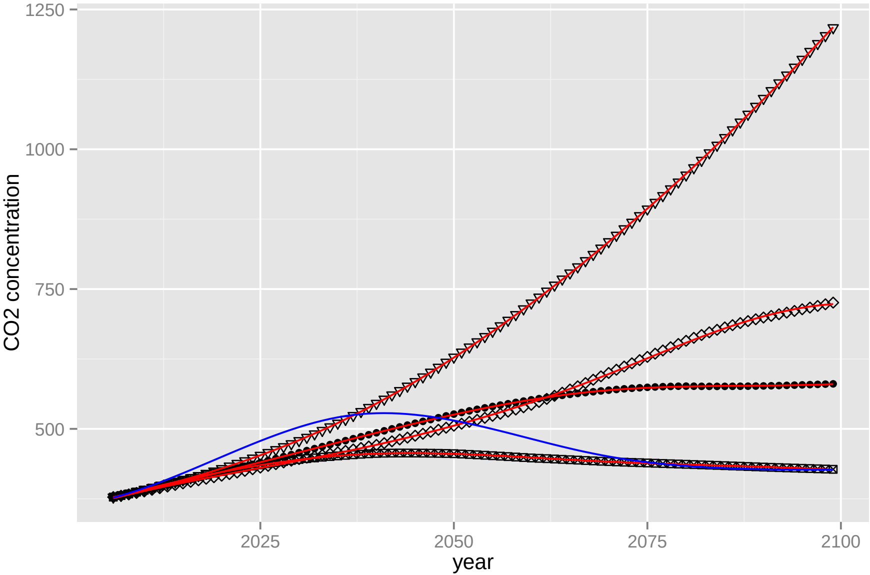

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Critical future research directions

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- Huge interdisciplinary challenges include:
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  - Including parameter uncertainty in CMIP based uncertainty/policy studies.
  - Understanding, modelling and quantifying model discrepancy.
  - Quantifying uncertainty in observations ($Z_H = Y_H + e_H$).
  - Decision support using all of the above.

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- ISBA - EnviBayes / RSS - ESS

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